MOHAMMedene / Med doudy 7C2 ERRAja

7C2 INTEGRALES

Exercice: 12

- Soit f est une fonction continue sur un itervalle ouvert I. Soient a et b des réels dans I.

1) Prouver que: $\int_a^b F(x)dx = \int_a^b F(a+b-x)dx$.

2) On pose $I = \int_0^{\pi/2} \frac{\cos^3 x\,dx}{\cos^3 x + \sin^3 x}$;

$J = \int_0^{\pi/2} \frac{\sin^3 dx}{\cos^3 x + \sin^3 x}$

Montrer que $I = J$

Calculer $I+J$. En deduire I et J

3) Calculer $K = \int_{\pi/6}^{\pi/3} (\sqrt{\cos x} - \sqrt{\sin x})dx$.

Solution

1) $A = \int_a^b f(a+b-x)\,dx$.

On pose $t = a+b-x$

$\Longrightarrow x = a+b-T$

$\begin{cases} x = a \Rightarrow a+b-a = b \Rightarrow T = b \\ x = b \Rightarrow a+b-b = b \Rightarrow T = b \end{cases}$

$\Longrightarrow dt = -dx \Rightarrow dx = -dt$

$A = \int_b^a f(t)(dt) = -\int_b^a f(x)dt = \int_a^b f(t)dt$

$$\boxed{\int_a^b f(a+b-x)dx = \int_a^b f(x)\,dx}$$

2) On pose $f(x) = \frac{\cos^3 x}{\cos^3 x + \sin^3 x}$

F est continue sur $[0;\frac{\pi}{2}]$

On prend $a = 0; b = \frac{\pi}{2}$;

On applique (1)

$\int_0^{\pi/2} f(\frac{\pi}{2}-x)\,dx = \int_0^{\pi/2} f(x)dx$

$F(\frac{\pi}{2}-x) = \frac{\cos^3(\frac{\pi}{2}-x)}{\cos^3(\frac{\pi}{2}-x) + \sin^3(\frac{\pi}{2}-x)}$

$= \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$

Donc: $\int_0^{\pi/2} f(\frac{\pi}{2}-x)\,dx = J$

Comme $\int_0^{\pi/2} f(x)\,dx = I$

On a alors $\{I = J\}$

http://maurimath.net/Espaceeleves.php

$$I+J=\int_0^{\pi/2}\frac{\cos^3 x+\sin^3 x}{\cos^3 x+\sin^3 x}\,dx=\int_0^{\pi/2}dx$$

$$I+J=[x]_0^{\pi/2}$$

$\boxed{I+J=\frac{\pi}{2}}$; On introduit que $\boxed{I=J=\frac{\pi}{4}}$

3) On pose :

$f(x)=\sqrt{\cos x}-\sqrt{\sin x}$

F est continue sur $[\frac{\pi}{6};\frac{\pi}{3}]$

$$F(a+b-x)=F\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)=F\left(\frac{\pi}{2}-x\right)=\sqrt{\cos(\frac{\pi}{2}-x)}-\sqrt{\sin(\frac{\pi}{2}-x)}=\sqrt{\sin x}-\sqrt{\cos x}$$

$$F(a+b-x)=-F(x)$$

- D'apres (1)

$$\int_{\pi/6}^{\pi/3}f(x)\,dx=\int_{\pi/6}^{\pi/3}f(x)\,dx$$

$$-\int_{\pi/6}^{\pi/3}f(x)\,dx=\int_{\pi/6}^{\pi/3}f(x)\,dx$$

$$-K=K\Rightarrow\boxed{K=0}$$

FIN