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EX 4:

1. a) $\lim_{n \rightarrow 0^+} f(x) = \lim_{n \rightarrow 0^+} 1 + x^3 - 3x^3 \ln x = 1$

$\Rightarrow \lim_{n \rightarrow 0^+} f(x) = f(0)$ donc g est continue en 0^+

continue en 0^+

* $\lim_{n \rightarrow +\infty} g(x) = \lim_{n \rightarrow +\infty} 1 + x^3 - 3x^3 \ln x$ F.I

$\lim_{x \rightarrow +\infty} (x^3 + 1 - 3x^3 \ln x) = -\infty$

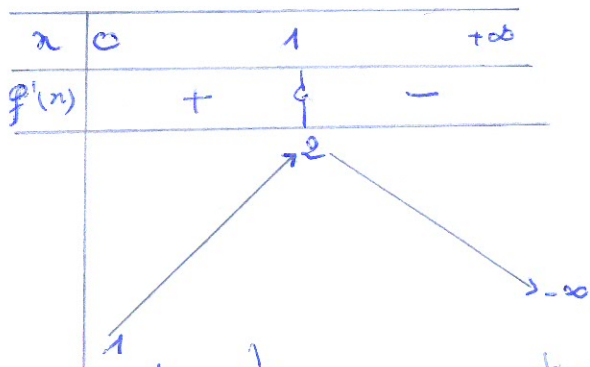
$g'(x) = 3x^2 - 9x^2 \ln x - 3x^2 = -9x^2 \ln x$

$g'(x) \geq 0 \forall x \in]0, 1[$

$g'(x) < 0 \forall x \in]1, +\infty[$

$f(1) = 1 + 1 - 0 = 2$

1) T.V



c) g est continue sur et strictement

decréissante sur $[1, +\infty[$

et change le signe donc l'équation $g(x) = 0$ admet une unique solution telle que $\alpha > 1$

$g(1) = 2 > 0$

$g(2) = -24 \ln 2 < 0$

x	1	α	$+\infty$
$g(x)$	+	0	-

2 a)

$\lim_{n \rightarrow 0^+} f(x) = \lim_{n \rightarrow 0^+} \frac{\ln x}{1+x^3} = \frac{-\infty}{1} = -\infty$

$\lim_{n \rightarrow +\infty} f(x) = \lim_{n \rightarrow +\infty} \frac{\ln x}{1+x^3}$ F.I

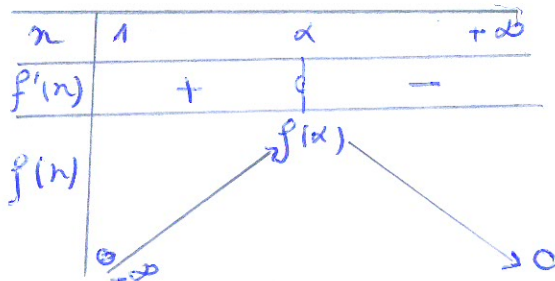
$\lim_{x \rightarrow +\infty} \frac{\ln x}{x} \times \left(\frac{x}{1+x^3} \right) = \lim_{x \rightarrow +\infty} 0 \times \left(\frac{1}{x^2} \right) = 0$

b) $f'(x) = \frac{1}{x} (1+x^3) - 3x^2 (\ln x)$

$f'(x) = \frac{1}{x} (1+x^3 - 3x^3 \ln x)$

$f'(x) = \frac{1}{x} (g(x))$, $\frac{g'(x)}{x(1+x^3)^2}$

T.V.



3 a)

f est continue sur $[1, +\infty[$ donc elle admet une primitive H dérivable sur $[1, +\infty[$

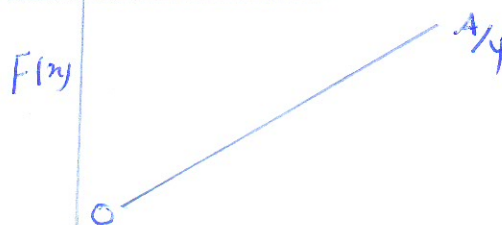
$F'(x) = H'(x) - H'(1)$

donc F est dérivable $F'(x) = f(x)$

pour tout $x > 1$, $f(x) > 0$

donc F est croissante

x	1	$+\infty$
$F(x)$	+	



Suite de exu.

b) pour tout $t > 1$.

$$1 < t^3 \leq 1+t^3 \leq (1+t)^3$$

$$\frac{1}{(1+t)^3} \leq \frac{1}{1+t^3} \leq \frac{1}{t^3}$$

$t > 1, \ln t > 0$

$$\frac{\ln t}{(1+t)^3} \leq \frac{\ln t}{1+t^3} \leq \frac{\ln t}{t^3}$$

$$\frac{\ln t}{(1+t)^3} \leq f(t) \leq \frac{\ln t}{t^3}$$

c) $\int_1^n \frac{\ln t}{t^3} dt$

on pose $\begin{cases} u(x) = \ln t \\ v(x) = t^3 \end{cases} \Rightarrow \begin{cases} u'(x) = \frac{1}{t} \\ v'(x) = -\frac{1}{2t^2} \end{cases}$

$$-\left[\frac{\ln t}{2t^2} \right]_1^n + \frac{1}{2} \int_1^n \frac{1}{t^3} dt$$

$$-\frac{\ln n}{2n^2} - \frac{1}{4} \left[\frac{1}{t^2} \right]_1^n$$

$$-\frac{\ln n}{2n^2} - \frac{1}{4n^2} + \frac{1}{4}$$

d)

$$\frac{1}{t(1+t)^3} = \frac{a}{t} + \frac{b}{1+t} + \frac{c}{(1+t)^2}$$

$$\frac{1}{(1+t)^3} = \frac{at(1+t) + bt(1+t) + ct}{t(1+t)^2}$$

$$\frac{1}{t(1+t)^2} = \frac{a+2at+at^2+bt+bt^2+ct}{t(1+t)^2}$$

$$\frac{1}{t(1+t)^2} = \frac{a+(a+b)t+(2a+b+c)t^2}{t(1+t)^2}$$

par identification

$$\begin{cases} a = 1 \\ a+b = 0 \\ 2a+b+c = 0 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -1 \\ c = -1 \end{cases}$$

$$\frac{1}{t(1+t)^2} = \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}$$

4.4)

$$\int_1^n \frac{\ln t}{(1+t)^3} dt \leq \int_1^n \frac{\ln t}{1+t^3} dt \leq \int_1^n \frac{\ln t}{t^3} dt$$

$$\int_1^n \frac{\ln t}{(1+t)^3} dt \leq f(t) \leq \int_1^n \frac{\ln t}{t^3} dt$$

on pose $J = \int_1^n \frac{\ln t}{(1+t)^3} dt$

$$\begin{cases} u(t) = \ln t \\ v'(t) = \frac{1}{(1+t)^3} \end{cases} \Rightarrow \begin{cases} u'(t) = \frac{1}{t} \\ v(t) = -\frac{1}{2(1+t)^2} \end{cases}$$

$$-\left[\frac{\ln t}{2(1+t)^2} \right]_1^n + \frac{1}{2} \int_1^n \frac{1}{t(1+t)^2} dt$$

$$\Rightarrow -\frac{\ln n}{2(1+n)^2} + \frac{1}{2} \left(\int_1^n \frac{1}{t} dt - \int_1^n \frac{1}{1+t} dt - \int_1^n \frac{1}{(1+t)^2} dt \right)$$

$$\Rightarrow -\frac{\ln n}{2(1+n)^2} + \frac{1}{2} \left(\left[\ln t \right]_1^n - \left[\ln(1+t) \right]_1^n + \left[\frac{1}{1+t} \right]_1^n \right)$$

$$-\frac{\ln n}{2(1+n)^2} + \frac{1}{2} \left(\ln n - \ln(n+1) + \ln e + \frac{1}{1+n} - \frac{1}{2} \right)$$

$$-\frac{\ln n}{2(1+n)^2} + \frac{1}{2} \left(\ln \left(\frac{n}{n+1} \right) + \frac{2\ln e - 1}{2} + \frac{1 - \frac{1}{2}}{1+n} \right)$$

$$-\frac{\ln n}{2(1+n)^2} + \frac{1}{2} \ln \left(\frac{n}{n+1} \right) + \frac{2\ln e - 1}{2} + \frac{1 - \frac{1}{2}}{2(n+1)}$$

Suite

$$\frac{-\ln x}{2(1+x)^2} = \frac{1}{2} \ln\left(\frac{x+1}{x}\right) - \frac{1-2\ln 2}{2} + \frac{1}{2(x+1)} - \frac{1}{4} \ll F(x) \ll \frac{1}{4} - \frac{1}{4x^2} - \frac{\ln x}{2x^2}$$

$$b) \lim_{n \rightarrow +\infty} \frac{-\ln n}{2(1+n)^2} = \lim_{n \rightarrow +\infty} \left(\frac{-\ln n}{n}\right) \times \left(\frac{n}{(1+n)^2}\right) = 0$$

$$\lim_{n \rightarrow +\infty} \frac{1}{2} \ln\left(1 + \frac{1}{n}\right) = \ln(1) = 0$$

$$\lim_{n \rightarrow +\infty} \frac{1}{2(n+1)} = \frac{1}{+2} = 0$$

$$\lim_{n \rightarrow +\infty} \frac{1}{4n} = 0$$

$$\lim_{n \rightarrow +\infty} \frac{-\ln n}{2n^2} = 0$$

$$\text{En fin } -\frac{1}{4} + \frac{1}{2} \ln 2 \ll l \ll \frac{1}{4}$$

1) Courbe

