2013N

Ex 4 :

1) $g(x) = \dfrac{3x^3 - 12x^2 + 19x - 10}{x^3 - 4x^2 + 5x}$

a) Cherchons a, b et c tel que $\forall\ x \in \mathbb{R}^*$:

$$g(x) = \frac{(x-1)(ax^2+bx+c)}{x(x^2-4x+5)}$$

Le T. H :

	3	-12	19	-10
1	X	3	-9	10
	3	-9	10	0

Donc : $a = 3$; $b = -9$ et $C = 10$.

$$\Rightarrow \quad g(x) = \frac{(x-1)(3x^2-9x+10)}{x(x^2-4x+5)} \quad \forall\ x \in \mathbb{R}^*$$

b) $g(x) = 0 \Rightarrow \begin{cases} x - 1 = 0 \Rightarrow x = 1 \\ 3x^2 - 9x + 10 = 0 \ : \ \Delta = 81 - 120 < 0 \end{cases}$

alors $3x^2 - 9x + 10$ positive $\forall\ x \in \mathbb{R}^*$.

$x(x^2 - 4x + 5) = 0 \Rightarrow \begin{cases} x = 0 \\ x^2 - 4x + 5 = 0 \Rightarrow \Delta = 16 - 20 < 0 \\ \Rightarrow x^2 - 4x + 5 \text{ est positive.} \end{cases}$

alors le signe de $g(x)$ est celui de $\dfrac{x-1}{x}$

Le tableau de signe de $\frac{x-1}{x}$:

x	$-\infty$	0		1	$+\infty$
x	$-$	$\vert 0$	$+$		$+$
$x-1$	$-$		$-$	0	$+$
$\frac{x-1}{x}$	$+$		$-$		$+$

Le signe de g

x	$-\infty$	0		1	$+\infty$
$g(x)$	$+$	$\Vert$	$-$	0	$+$

2)_ $f(x) = 3x - 3 + \ln\left(\frac{x^2 - 4x + 5}{x^2}\right) \quad \forall\, x \in \mathbb{R}^*$.

2.a)_ $\lim\limits_{x \mapsto 0} f(x) = 0 - 3 + \ln\left(\frac{5}{0^+}\right) = -3 + \infty = +\infty$.

$\boxed{X = 0}$ A.V pour (C) au voisinage de 0.

2.b)_ . $\lim\limits_{x \mapsto +\infty} f(x) = +\infty - 3 + \ln(1) = +\infty$.

. $\lim\limits_{x \mapsto -\infty} f(x) = -\infty - 3 + \ln(1) = -\infty$.

1.c) -
$\lim_{x \to \mp\infty} f(x) - (3x-3) = \lim_{x \to \pm\infty} \ln\left(\frac{x^2-4x+5}{x^2}\right) = 0$

Donc : $D : y = 3x-3$ est un A.O pour (C) au voisinage de (∞).

$X = 0$ A.V pour (C).

$f(x) - (3x-3) = \ln\left(\frac{x^2-4x+5}{x^2}\right)$

$\frac{x^2-4x+5}{x^2} \geq 1$ ssi $f(x) - (3x-3) \geq 0$ Car : $\ln\left(\frac{x^2-4x+5}{x^2}\right) \geq 0$

$\frac{x^2-4x+5}{x^2} \geq 1$

$x^2-4x+5 \geq x^2 \Rightarrow -4x+5 \geq 0 \Rightarrow 4x \leq 5 \Rightarrow$

$\boxed{x \leq \frac{5}{4}}$

P.R :

x	$-\infty$	0	$5/4$	$+\infty$
$f(x) - y_0$	+	‖ +	0	−
P.R	C/D	‖ C/D		D/C

3.a) - $f'(x) = 3 + \frac{\left(\frac{x^2-4x+5}{x^2}\right)'}{\frac{x^2-4x+5}{x^2}} =$

$= 3 + \frac{[(2x-4)\cdot x^2 - 2x(x^2-4x+5)]\cdot x^2}{x^4\cdot(x^2-4x+5)}$

$= 3 + \frac{x^3(2x^2-4x-2x^2+8x^1-10)}{x^4\cdot(x^2-4x+5)}$

$$f'(x) = 3 + \frac{(2x^2 - 4x - 2x^2 + 8x - 10)}{x(x^2 - 4x + 5)}$$

$$= 3 + \frac{4x - 10}{x(x^2 - 4x + 5)}$$

$$= \frac{3x^3 - 12x^2 + 15x + 4x - 10}{x(x^2 - 4x + 5)}$$

$$= \frac{3x^3 - 12x^2 + 19x - 10}{x(x^2 - 4x + 5)}$$

$$\boxed{f'(x) = g(x)}$$

Le T.V de f :

x	$-\infty$		0		1		$+\infty$
$f'(x)$		$+$	||	$-$	0	$+$	
$f(x)$	$-\infty$	$\nearrow$	$+\infty$ || $+\infty$	$\searrow$	$\ln 2$	$\nearrow$	$+\infty$

3.b)- f est continue sur $\mathbb{R}^*$ f change de signe une seul fois dans $\mathbb{R}^*$ alors la solution (α) de l'equation $f(x) = 0$ est unique. Cherchons une encadrement de α.

- $f(-1) = -6 + \ln 10 \simeq -3,69$ $\Rightarrow f(0) \cdot f(-0,5) < 0$
- $f(-0,5) = -1,13$.
- $\lim\limits_{x \to 0} f(x) = +\infty$

$\Rightarrow \boxed{-0,5 < \alpha < 0}$

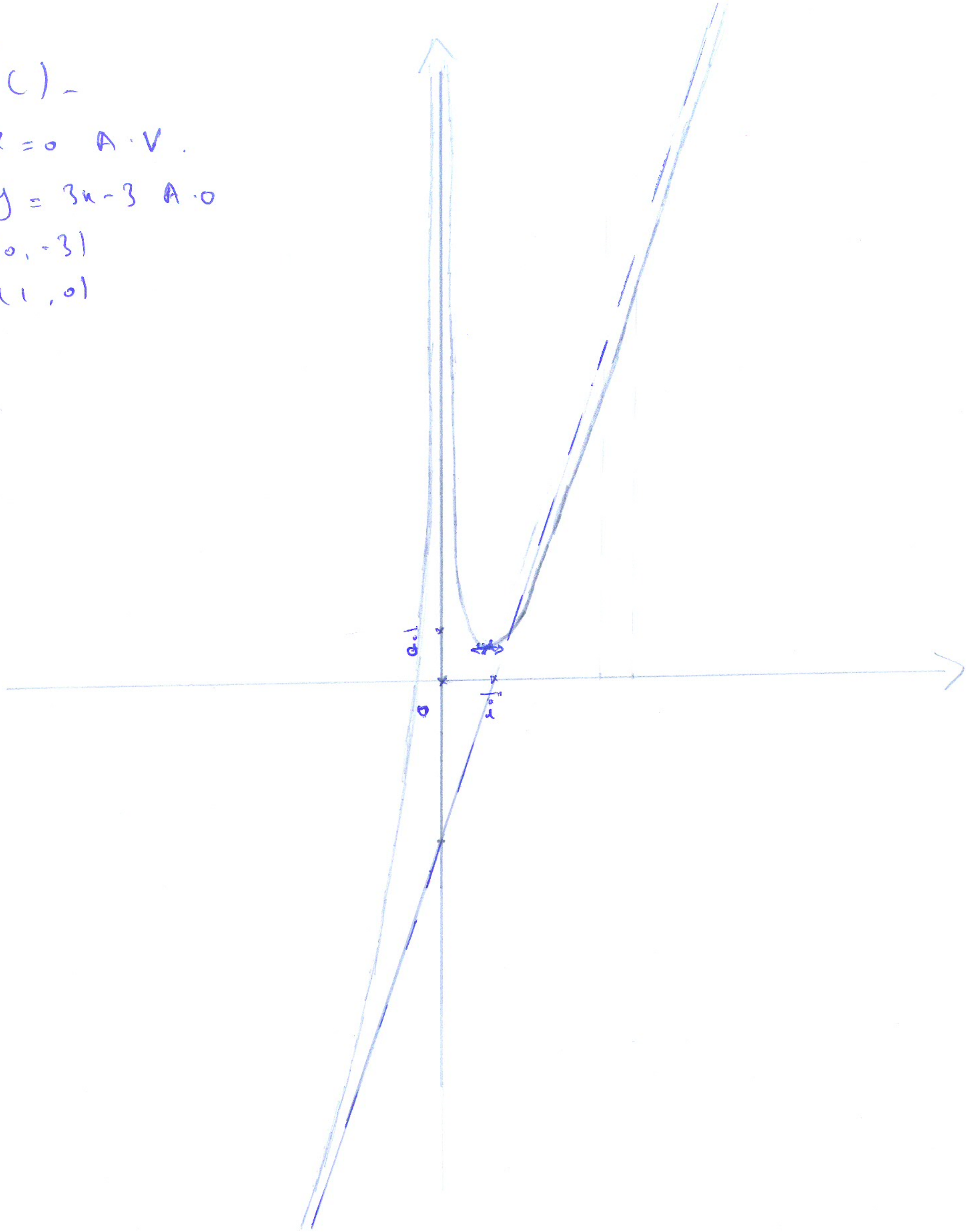
3. c) -

$x = 0$ A.V.

$y = 3x - 3$ A.O

$(0, -3)$

$(1, 0)$



4.) $S = \int_3^{2+\sqrt{3}} f(x) - (3x-3)\, dx$.

a)- $2\left(1 + \frac{2x-4}{x^2-4x+5} - \frac{1}{1+(x-2)^2}\right) =$

$= 2 + \frac{4x-8}{x^2-4x+5} - \frac{2}{1+x^2-4x+4}$

$= \frac{2x^2-8x+10 + 4x-8 - 2}{x^2-4x+5}$

$= \frac{2x^2-4x}{x^2-4x+5}$

c)- on pose : $X = 2 + \tan t \quad \forall t \in [0, \frac{\pi}{2}[$

$B = \int_2^{2+\sqrt{3}} \frac{1}{1+(x-2)^2} dx$

. $2+\sqrt{3} = 2 + \tan t \Rightarrow \tan t = \sqrt{3} \Rightarrow t = \frac{\pi}{3}$.

. $2+1 = 2+\tan t \Rightarrow \tan t = 1 \Rightarrow t = \frac{\pi}{4}$.

$X = 2+\tan t \Rightarrow dx = (1+\tan^2 t)\, dt$

$B = \int_{\frac{\pi}{4}}^{\pi/3} \frac{1}{1+(2+\tan t-2)^2}(1+\tan^2 t)\, dt = \int_{\pi/4}^{\frac{\pi}{3}} \frac{1}{1+\tan^2 t}(1+\tan^2 t)\, dt$

$= [t]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{3} - \frac{\pi}{4} \Rightarrow \boxed{B = \frac{\pi}{12}}$

d) - $J = \int_3^{2+\sqrt{3}} \ln(x^2-4x+5)\,dx$

on pose :

$$\begin{cases} u(x) = \ln(x^2-4x+5) \\ v'(x) = 1 \end{cases} \qquad \begin{cases} u'(x) = \dfrac{2x-4}{x^2-4x+5} \\ v(x) = x \end{cases}$$

$$J = -\int_3^{2+\sqrt{3}} \frac{x(2x-4)}{x^2-4x+5}\,dx + \Big[x\ln(x^2-4x+5)\Big]_3^{2+\sqrt{3}}$$

$$= (2+\sqrt{3})\ln(4+4\sqrt{3}+3-8-4\sqrt{3}+5)$$
$$- 3\ln(9-12+5) - 2\int_3^{2+\sqrt{3}} 1 + \frac{2x-4}{x^2-4x+5} - \frac{1}{1+(x-2)^2}\,dx$$

$$= (2+\sqrt{3})\ln(4) - 3\ln(2) - 2\int_3^{2+\sqrt{3}} 1\,dx - 2\int_3^{2+\sqrt{3}} \frac{2x-4}{x^2-4x+5} - \frac{1}{1+(x-2)^2}\,dx$$

$$= 2(2+\sqrt{3})\ln(2) - 3\ln 2 - 2\Big[x\Big]_3^{2+\sqrt{3}} - 2\Big[\ln(x^2-4x+5)\Big]_3^{2+\sqrt{3}}$$
$$+ 2\int_3^{2+\sqrt{3}} \frac{1}{1-(x-2)^2}\,dx$$

$$= (1+2\sqrt{3})\ln(2) - 2(2+\sqrt{3}-3) - 2(\ln(4) - 1\ln(2))$$
$$+ 2B$$

$$= (1+2\sqrt{3})\ln(2) - 2(-1+\sqrt{3}) - 2\cdot\ln 2 + 2B$$

$$= 2-2\sqrt{3} + \frac{\pi}{6} + (-2\ln 2) + (1+2\sqrt{3})\ln(2)$$

$$\Rightarrow \boxed{J = (-1+2\sqrt{3})\ln(2) + 2 - 2\sqrt{3} + \frac{\pi}{6}}$$

$$K = 2\int_3^{2+\sqrt{3}} \ln(x)\,dx$$

on pose : $\begin{cases} U(x) = \ln(x) \\ V'(x) = 1 \end{cases} \Rightarrow \begin{cases} U'(x) = \frac{1}{x} \\ V(x) = x \end{cases}$

$$K = 2\left[x\ln(x)\right]_3^{2+\sqrt{3}} - 2\int_3^{2+\sqrt{3}} dx$$

$$= 2\left((2+\sqrt{3})\ln(2+\sqrt{3}) - 3\ln(3)\right) - 2\left[x\right]_3^{2+\sqrt{3}}$$

$$= (4+2\sqrt{3})\ln(2+\sqrt{3}) - 6\ln(3) + (-2-\sqrt{3}+3)2$$

$$\boxed{K = (4+2\sqrt{3})\ln(2+\sqrt{3}) - 6\ln 3 + 2 - \sqrt{3}}$$

$$S' = +\int_3^{2+\sqrt{3}} (3x-3) - f(x)\,dx = -\int_3^{2+\sqrt{3}} \ln\left(\frac{x^2-4x+5}{x^2}\right) dx$$

$$= -\int_3^{2+\sqrt{3}} \ln(x^2-4x+5)\,dx + \int_3^{2+\sqrt{3}} \ln(x^2)\,dx$$

$$= -\int_3^{2+\sqrt{3}} \ln(x^2-4x+5)\,dx + 2\int_3^{2+\sqrt{3}} \ln(x)\,dx$$

$$= -J + K = -\left[(-1+2\sqrt{3})\ln(2) + 2 - 2\sqrt{3} + \frac{\pi}{6}\right)$$

$$+ (4+2\sqrt{3})\ln(2+\sqrt{3}) - 6\ln(3) + 2 - \sqrt{3}$$

$$\simeq 1{,}006 \cdot \boxed{U.A}$$