Bac 2015 SN :

Ex 1 :

1)_ $P(z) = z^3 - (11+6i)z^2 + (28+38i)z - 12 - 6i$

1.a)_ $P(3) = 3^3 - (11+6i)(3)^2 + (28+38i)3 - 12 - 6i$

$= 27 - 99 - 54i + 84 + 114i - 12 - 6i$

$= 12\times1 - 12 + 144i - 144i$

$= 0$

$\Rightarrow P(3) = 0$

T.H

	1	$-(11+6i)$	$28+38i$	$-12-60i$
3	$\times$	3	$-24-18i$	$12+60i$
	1	$-8-6i$	$4+20i$	0

$a = -(8+6i)$, $b = 4+20i$.

$P(z) = (z-3)(z^2 - (8+6i)z + 4 + 20i)$

1.b)_ $P(z) = 0 \Leftrightarrow \begin{cases} z - 3 = 0 \Rightarrow z_0 = 3 . \\ z^2 - (8+6i)z + 4 + 20i = 0 \end{cases}$

$z^2 - (8+6i)z + 4 + 20i = 0$

$\Delta = 64 + 96i - 36 - 16 - 80i$

$= 12 + 16i$

$= 4^2 + 2i\times4\times2 + (2i)^2$

$= (4+2i)^2 \Rightarrow \delta = 4 + 2i$.

$$z_1 = \frac{8+6i-4-2i}{2} = \frac{4+4i}{2} = 2+2i$$

$$z_2 = \frac{8+6i+4+2i}{2} = \frac{12+8i}{2} = 6+4i$$

$$S' \left\{ 3,\ 2+2i,\ 6+4i \right\}.$$

c) on a : $\operatorname{Im} z_A < \operatorname{Im} z_B < \operatorname{Im} z_C$

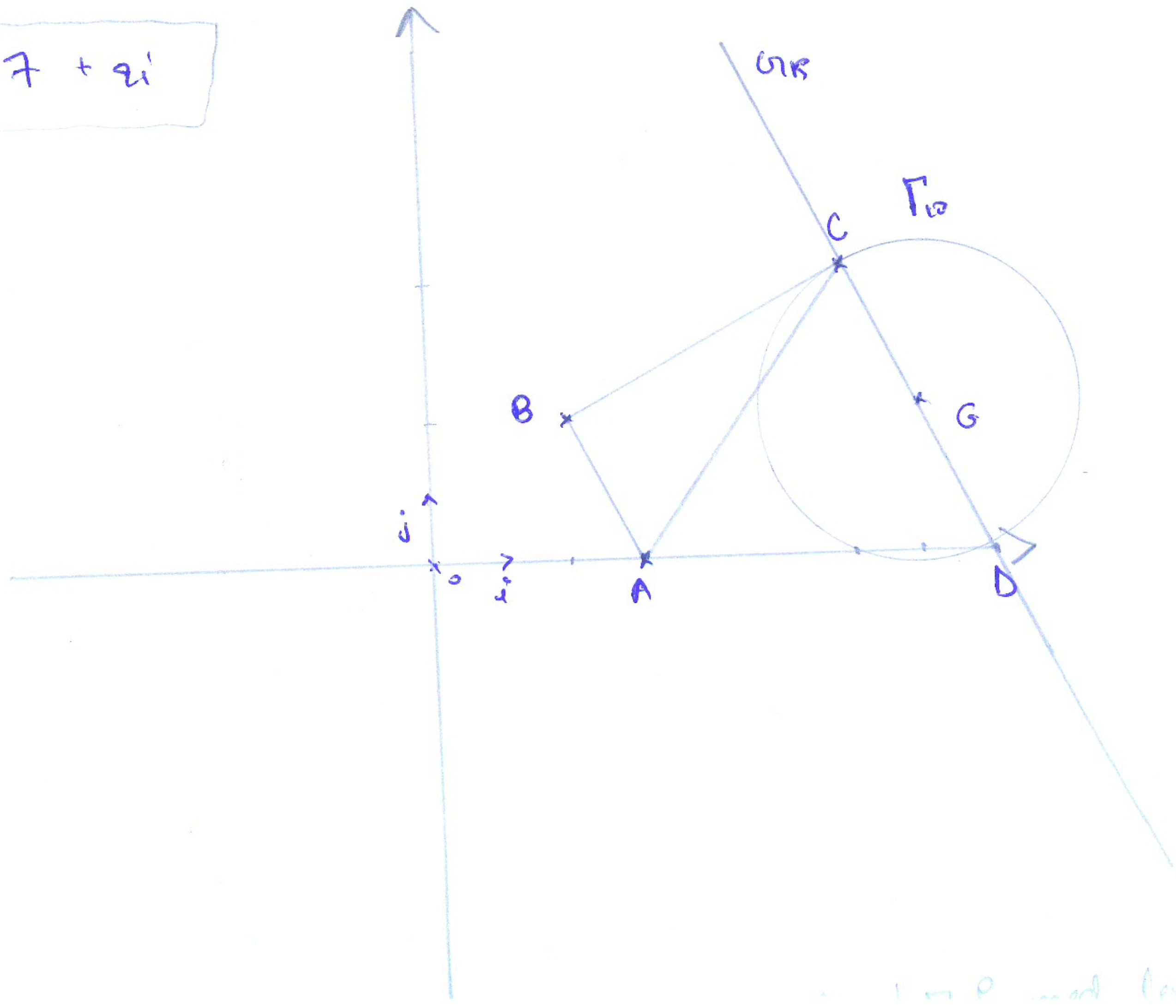
$\operatorname{Im} 3 < \operatorname{Im}(2+2i) < \operatorname{Im}(6+4i)$

$z_A = 3$, $z_B = 2+2i$, $z_C = 6+4i$.

$$G = \text{bar} \begin{array}{c|c|c} A & B & C \\ \hline 2 & -2 & 2 \end{array}$$

$$z_G = \frac{2z_A - 2z_B + 2z_C}{2} = \frac{6-4-4i+12+8i}{2} = \frac{14+4i}{2}$$

$$\boxed{z_G = 7+2i}$$



1)- $\forall K \in \mathbb{R} \setminus \{2\}$;

$$\overrightarrow{MM'} = 2\overrightarrow{MA} - 2\overrightarrow{MB} + (3-K)\overrightarrow{MC}$$

$$z' - z = 2(z_A - z) - 2(z_B - z) + (3-K)(z_C - z)$$

$$\Rightarrow z' = z + 2z_A - \cancel{2z} - 2z_B + \cancel{2z} + 3z_C - Kz_C - 3z + Kz$$

$$= (1-3+K)z + 2z_A - 2z_B + z_C(3-K)$$

$$\boxed{z' = (K-2)z + 2z_A - 2z_B + z_C(3-K)}$$

L'application f_K est une translation ssi $K-2=1 \Rightarrow K=3$

• Si $K=3$ f_3 est une translation cherchon $\vec{u}$.

$$2z_A - 2z_B + 0 = 2\times 3 - 2(2+2i)$$

$$= 6 - 4 - 4i$$

$$= 2 - 4i$$

$\Rightarrow f_3 = t_{\vec{u}}$ tel que $\vec{u}\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

2)- Comme $K-2 \in \mathbb{R} \setminus \{0,1\}$ alors f_K admet une unique points invariant Ω_K.

tel que f_K est homothetie de centre Ω_K et de rapport $K-2$.

$$z_{\Omega_K} = \frac{2\times 3 - 2(2+2i) + (3-K)z_C}{1-K+2} = \frac{6-4-4i+(3-K)z_C}{3-K}$$

$$z_{\Omega_K} = \frac{2-4i+(3-K)(6+4i)}{3-K} = \frac{2-4i+18-6K+12i-4iK}{3-K}$$

$$= \frac{20-6K+8i-4iK}{3-K} = \frac{20-6K}{3-K} + \frac{i(8-4K)}{3-K}$$

c). $\begin{cases} x_{\omega_K} = \dfrac{20 - 6K}{3 - K} = 6 + \dfrac{2}{3 - K} \\ y_{\omega_K} = \dfrac{8 - 4K}{3 - K} = 4 - \dfrac{4}{3 - K} \end{cases}$

$2x + y = 16 \Rightarrow \boxed{2x + y - 16 = 0}$

on remarque que les coordonné de (C) verifie l'equation.

• si $\boxed{y = 0} \Rightarrow \boxed{x = 8}$ on pose $D(0, 8)$.

Alors ω_K decrit la droit (CD).

ω_1 : $x_{\omega_1} = 6 + \dfrac{2}{2} = 6 + 1 = 7$.

$y_{\omega_1} = 4 - \dfrac{4}{2} = 4 - 2 = 2$.

$\Rightarrow \boxed{\omega_1 \equiv G}$

II). Pour $K = 1$: $z' = -z + 2z_A - 2z_B + 2z_C$

$z' = -z + z_G$

Comme le coefficient de z est -1 alors f_1 est une symetrie central.

$R = \text{bar}$

A	M	M'
1	1	1

Comme f_1 est un symetrie Central de Centre $\omega_1 = G$, alors $G = M * M'$,

$R = \text{bar}$

A	G
1	2

c G est fixe $\forall M \in P$
A est fixe $\forall M \in P$ $\Rightarrow$

R est fixe car $\boxed{\overrightarrow{AR} = \dfrac{2}{3} \overrightarrow{AG}}$

3)- $\varphi(M) = 2MA^2 - 2MB^2 + 2MC^2$

$\varphi(M) = m \quad . \quad m \in \mathbb{R}$.

a)- $GA^2 = |z_A - z_G|^2 = |3 - 7 - 2i|^2$

$= |-4 - 2i|$

$= 16 + 4$

$= 20$.

$GB^2 = |z_B - z_G|$

$= |2 + 2i - 7 - 2i|$

$= |-5|^2$

$= 25$.

$GC^2 = |z_C - z_G|^2$

$= |6 + 4i - 7 - 2i|^2$

$= |-1 + 2i|^2$

$= 1 + 4$

$= 5$.

$\varphi(G) = 2GA^2 - 2GB^2 + 2GC^2$

$= 40 - 50 + 10$

$= 0$.

$\varphi(M) = 2MA^2 + \varphi(G)$

$= 2MA^2 = m \Rightarrow \boxed{M = \sqrt{\frac{m}{2}}}$.

si $m < 0 \Rightarrow \emptyset$.

Si $m = 0 \Rightarrow M \equiv G$

si $m > 0 \Rightarrow M$ decrit $\mathcal{C}(G, \sqrt{\frac{m}{2}}) = \Gamma_m$

b)- si $m = 10$

$\Gamma_m = \mathcal{C}_{(G, 10)}$.